New Dissipation Term and Self-Similar Relationships for Hasselmann Equation

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LONG-TERM GOALS

The theory of weak turbulence is the cornerstone of modern wave-predicting models. It is based on the Hasselmann equation for the wave action spectrum $N(\omega,\theta)$:

$$\frac{\partial N}{\partial t} + \frac{\partial \omega}{\partial k}\frac{\partial N}{\partial r} = S_{nl} + S_F, \qquad S_{nl} = S_{in} + S_{diss} \tag{1}$$

Here S_{nl} is the nonlinear wave interaction term, S_{in} is the air-wave coupling term, and S_{diss} is responsible for the white-capping dissipation.

In spite of tremendous efforts invested in numerical studies of the ***Eq.1***, the progress in development of fast and accurate prognostic models is slow. Just as an illustration, the scatter of S_{in} in different models of exceeds its mean value up to the factor of 4 !. As a result, the major verification tool for wave-prediction models -- hind-casting of ocean surface wave events -- consists in "tuning-up" of coefficients in front of S_{in} and S_{diss} terms to achieve reasonable correspondence of numerical simulation results and measurable field parameters.

Our long-term goal is improvement of surface wave – prediction models through:

- Formulation of new S_{in} and S_{diss} terms, which can be used in wave-predicting models without necessity of empirical tuning-up to achieve adequate description o the reality
- Search of the universal self-similar properties of Hassellmann equation (1), describing the behavior of wave turbulence in terms of limited number of intrinsic physical quantities, such as: wave energy spectral flux, characteristic wave frequencies and universal self-similar parameters.

OBJECTIVES

Our first objective was to concentrate on the study of S_{diss} term through direct numerical comparison of surface wave turbulence in the frame of full 3D equations with Hasselmann equation, for the sitiation when the wind is absent, i.e. pure swell setup.

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The second objective was to compare the comprehensive experimental set of data, obtained over almost last 50 years of field observations with the result of analytical and numerical results of study of Hasselmann equation.

APPROACH

For achievement of our objectives we used the following two numerical tools:

- ***S****tatistical* ***C****omputational* ***W****ave* ***T****ank* (***SCWT***) numerically simulates non-stationary Hasselmann equation:

$$\frac{\partial N}{\partial t} + \frac{\partial \omega}{\partial k}\frac{\partial N}{\partial x} = S_{nl} + S_F \qquad S_{nl} = S_{in} + S_{diss} \tag{3}$$

  The algorithm is updated Resio-Tracy code, which is two orders of magnitude faster then the original version. The new version is stable. We used the grid in ***K***-plane containing 36 angular versus 71 frequency modes. For the wind velocity $u_0 = 10m/\sec$ it takes eight hours of computational time to model two days evolution of the wind-driven sea in one spatial point.

- ***D****ynamical* ***C****omputational* ***W****ave* ***T****anks* (***DCWT***) follows not only the wave surface elevation, but also the wave phases and is based on the truncated expansion of the Hamiltonian into the series by powers of the surface elevation:

$$\begin{aligned} \eta_t &= \hat{k}\psi - (\nabla(\eta\nabla\psi)) - \hat{k}[\eta\hat{k}\psi] + \hat{k}(\eta\hat{k}[\eta\hat{k}\psi]) + \tfrac{1}{2}\Delta[\eta^2\hat{k}\psi] + \tfrac{1}{2}\hat{k}[\eta^2\Delta\psi] \\ \psi_t &= -g\eta + \sigma\Delta\eta - \tfrac{1}{2}[(\nabla\psi)^2 - (\hat{k}\psi)^2] - [\hat{k}\psi]\hat{k}[\eta\hat{k}\psi] - [\eta\hat{k}\psi]\Delta\psi \end{aligned} \tag{4}$$

  where $\eta(x,y)$ is water surface elevation and $\psi(x,y)$ is velocity potential on the surface of the liquid, g and σ are gravity acceleration and surface tension coefficient. Action of the operator $\hat{k}$ is defined as $\hat{k}\psi = \frac{1}{2\pi}\int k\psi_{\vec{k}} e^{-i\vec{k}\vec{r}} dk_x dk_y$, where $k = \sqrt{k_x^2 + k_y^2}$ is the absolute value of the wave number. The simulation of ***DCWT*** was performed in real-space domain $[2\pi \times 2\pi]$ on the grid 512x4096 points.

We hired two contractors groups of applied mathematicians and physical oceanographers in Moscow:

1. Group in the Shirshov Institute of Oceanology, Moscow, Russia. A head of the group is Dr. Sergei Badulin.

2. Group in the Landau Institute for Theoretical Physics, Moscow, Russia. A head of the group is Dr. Alexander Dyachenko.

The following are key individuals involved into research:

Principals:

- **Zakharov V.E.** – general supervision
- **Pushkarev A.N**.– Hasselmann and 2D dynamical equations

Contractors: **Group 1:**

- **Badulin S.I.** -- time-limited S_{nl} code for Hasselmann equation
- **Geogdjaev V.V.** -- time-limited S_{nl} code for Hasselmann equation

Group 2:

- **Dyachenko A.I.–** 1D exact dynamical equations
- **Korotkevich A.O.** – 2D dynamical equations
- **Prokofiev A**.– exact dynamical equations

WORK COMPLETED

1. Numerical comparison of swell evolution in frame of two models: ***SCWT*** and ***DCWT***.

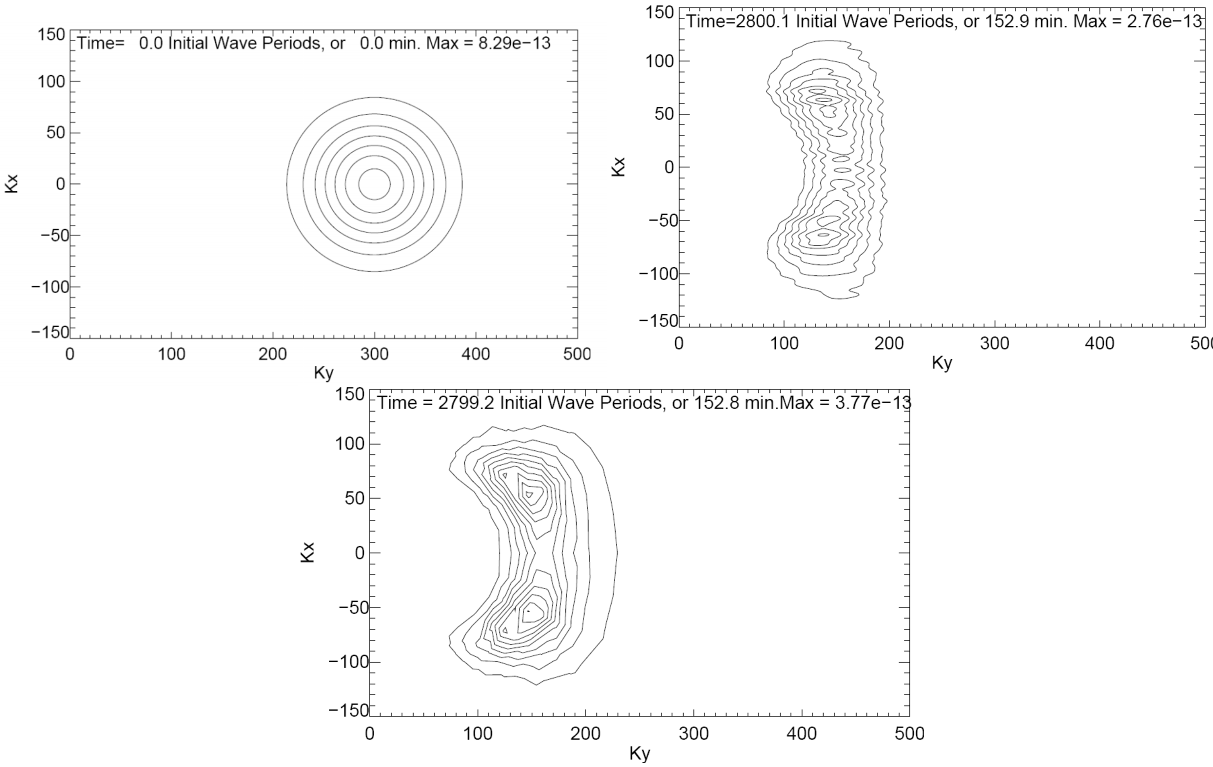
2. Comparison of the comprehensive set of 20 dependencies, obtained over last 50 years from field observations, with analysis of Hasselmann equation and numerical simulation of ***SCWT*** model.

RESULTS

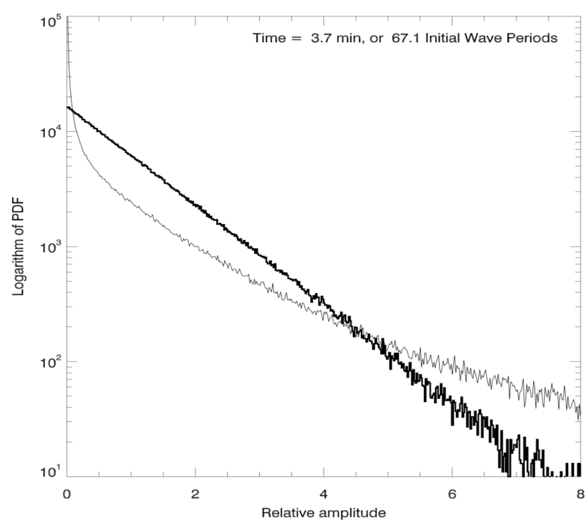
The first major result is confirmation of the theory of weak turbulence through direct comparison of ***SCWT*** and ***DCWT*** models. Comparison was performed for pure swell (no wind input, $S_{in} = 0$) situation. Similarity was observed in spectral peak downshift and bi-modality of the spectrum (see ***Fig.2***). Probability of distribution function of wave amplitudes exhibits Gaussian distribution (see ***Fig.3***). We compare ***SCWT*** and ***DCWT*** models for dissipation terms S_{diss}: artificial viscosity, *WAM cycle 3, WAM cycle 4* and new dissipation terms. The result of comparison of total energy behavior is presented on ***Fig.4***. It has been learned that none of the dissipation terms except new one give good correspondence in total energy behavior in time. The formula for new dissipation term is the following:

$$S_{diss} = C_{ds}\tilde{\omega}\frac{k}{\tilde{k}}\left((1-\delta)+\delta\frac{k}{\tilde{k}}\right)\left(\frac{\tilde{S}}{\tilde{S}_{pm}}\right)^{p} n_k$$

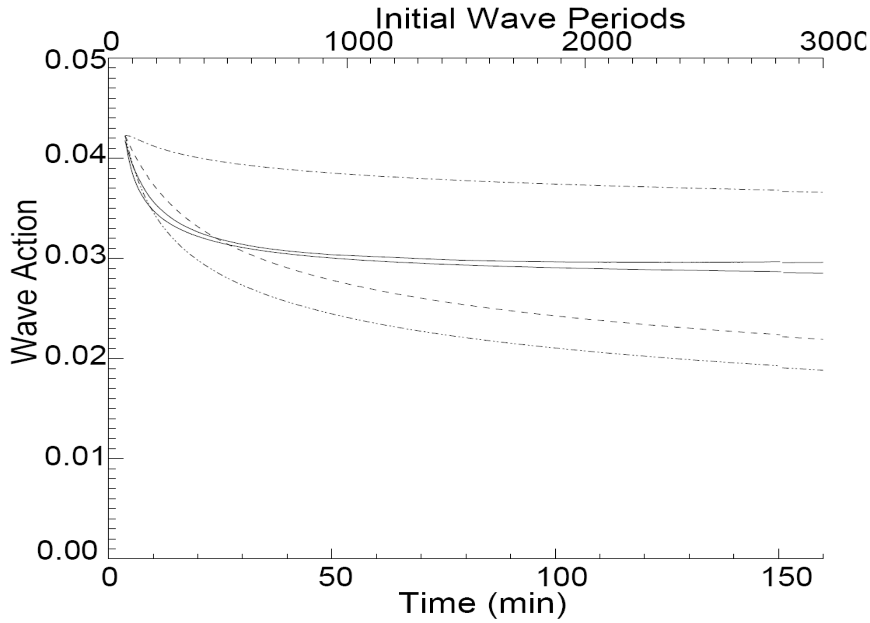
$$C_{ds} = 1.00\times 10^{-6},\ \ \delta = 0,\ \ p = 12 \qquad (5)$$



[Fig.2 Original spectrum (first picture) down-shifts in DCWT (second picture) and SCWT (third picture) models]



[Fig.3 Probability distribution function of wave amplitudes is approximated by straight line, which confirms Gaussian distribution law]



[Fig.4 Total energy behavior for DCWT and SCWT models with different dissipation terms. The best correspondence reached for new dissipation term]

The second major result is in obtaining of an asymptotic weakly turbulent relation for the total energy ε and a characteristic wave frequency ω_*:

$$\frac{\varepsilon\omega_*^4}{g^2} = \alpha_{ss}\left(\frac{\omega_*^3\, d\varepsilon/dt}{g^2}\right)^{1/3} \tag{6}$$

The self-similarity parameter α_{ss} was found in the numerical duration-limited experiments and was shown to be naturally varying in a relatively narrow range, being dependent on the energy growth scale only. The analytical and numerical conclusions have been verified versus known field dependencies for wave energy growth and peak frequency down-shift. A comprehensive set of more than 20 dependencies, obtained over almost 50 years of field observations, was analyzed. They demonstrate that the weakly turbulent law has a general value and describes the wave evolution well, apart from the earliest and full wave development stages when nonlinear transfer competes with wave input and dissipation.

IMPACT/APPLICATIONS

Potential future impact for obtained results consists in:

- Using correct dissipation term (5) in operational wave-prediction models
- Using relationship (6) for surface wave turbulence parameters forecasts.

RELATED PROJECTS

US Army Corps of Engineers ***MORPHOS*** project. Our contribution consists in continuation of the development of new forms of wave-breaking within the context of nonlinear energy fluxes in a wind – wave spectrum.

PUBLICATIONS

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3. V.E. Zakharov, A.O. Korotkevich, A. Pushkarev and D. Resio, *Coexistence of Weak and Strong Wave Turbulence in a Swell Propagation,* accepted for publication in Phys.Rev.Letters, 2007
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5. Report on the *10th International Workshop on Wave Hindcasting and Forecasting & Coastal Hazard Symposium*, Oahu, HI. Reporter V. Zakharov